Social Welfare and Transition Dynamics

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Macroeconomics III

Welfare Analysis and Policy Changes

Welfare Computation

- We may be interested in the "desirability" of a reform.
- Some agents lose, others gain.

Welfare Computation

- We may be interested in the "desirability" of a reform.
- Some agents lose, others gain.
- Utility not a cardinal measure. How to aggregate?

Welfare Comparison in Stationary Equilibrium

Assume agents solve

$$\max \left\{ \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \frac{c_t^{1-\gamma}}{1-\gamma} \right\}$$

- There is some stochastic state vector $\mathbf{x}$ and stationary policy rules.
- If reform is not enacted, the consumption function is $c(\mathbf{x})$.
- If reform is enacted, the consumption function becomes $\hat{c}(\mathbf{x})$.
- What is the consumption equivalence between the two worlds?

Welfare Comparison in Stationary Equilibrium II

Assume in $t = 0$, agents are distributed according to $\lambda(\mathbf{x})$. We are interested in the ω^U that solves

$$\int E_0 \sum_{t=0}^{\infty} \beta^t U\left([1+\omega^U]c(\mathbf{x})\right) d\lambda(\mathbf{x}) = \int E_0 \sum_{t=0}^{\infty} \beta^t U\left(\hat{c}(\mathbf{x})\right) d\hat{\lambda}(\mathbf{x}),$$

where $\hat{\lambda}(\mathbf{x})$ is the stationary distribution that occurs when the policy is enacted. The welfare measure is based on an unborn household (veil of ignorance).

Welfare Comparison in Stationary Equilibrium III

$$\int E_0 \sum_{t=0}^{\infty} \beta^t \frac{\left([1+\omega^U] c(\mathbf{x})\right)^{1-\gamma}}{1-\gamma} d\lambda(\mathbf{x}) = \int E_0 \sum_{t=0}^{\infty} \beta^t \frac{\hat{c}(\mathbf{x})^{1-\gamma}}{1-\gamma} d\hat{\lambda}(\mathbf{x})$$

$$[1+\omega^U]^{1-\gamma} \int E_0 \sum_{t=0}^{\infty} \beta^t \frac{c(\mathbf{x})^{1-\gamma}}{1-\gamma} d\lambda(\mathbf{x}) = \int E_0 \sum_{t=0}^{\infty} \beta^t \frac{\hat{c}(\mathbf{x})^{1-\gamma}}{1-\gamma} d\hat{\lambda}(\mathbf{x})$$

$$[1+\omega^U]^{1-\gamma} \int V_0(\mathbf{x}) d\lambda(\mathbf{x}) = \int \hat{V}_0(\mathbf{x}) d\hat{\lambda}(\mathbf{x})$$

$$\omega^U = \left(\frac{\int \hat{V}_0(\mathbf{x}) d\hat{\lambda}(\mathbf{x})}{\int V_0(\mathbf{x}) d\lambda(\mathbf{x})}\right)^{\frac{1}{1-\gamma}} - 1$$

Welfare Comparison in Stationary Equilibrium IV

$$\omega^U = \left(\frac{\int \hat{V}_0(\mathbf{x}) d\hat{\lambda}(\mathbf{x})}{\int V_0(\mathbf{x}) d\lambda(\mathbf{x})} \right)^{\frac{1}{1-\gamma}} - 1$$

- $\omega^U > 0$: The winners of the reform could compensate the losers.
- There maybe more losers than winners. You can compute it.
- Absent lump-sum taxation, redistribution may not be feasible.

Decomposing Welfare Change

Welfare is different because mean consumption differs:

$$[1+\omega^L]\int c(\mathbf{x})d\lambda(\mathbf{x}) = \int \hat{c}(\mathbf{x})d\hat{\lambda}(\mathbf{x})$$

$$[1+\omega^L]C = \hat{C}$$

Welfare is different because consumption volatility differs:

$$V(\bar{c}(\mathbf{x})) = V(c_{0:\infty}(\mathbf{x}))$$

Average certainty equivalence:

$$\bar{C} = \int \bar{c}(\mathbf{x})d\lambda(\mathbf{x})$$

Cost of uncertainty:

$$p^{unc} = 1 - \left(\frac{V(\bar{C})}{V(C)}\right) \quad \omega^{unc} = \frac{1-\hat{p}^{unc}}{1-p^{unc}} - 1$$

Decomposing Welfare Change II

Welfare is different because consumption inequality:

$$p^{inq} = 1 - \left(\frac{\int V(\bar{c}(\mathbf{x})) d\lambda(\mathbf{x})}{V(\bar{C})} \right) \quad \omega^{inq} = \frac{1 - \hat{p}^{inq}}{1 - p^{inq}} - 1$$

Flodén (2001) shows aggregation hold:

$$\omega^{U} + 1 = (1 + \omega^{L})(1 + \omega^{unc})(1 + \omega^{inq})$$

Welfare with Transition Dynamics

So far, we compared welfare in old and new stationary equilibrium. Yet getting to new equilibrium takes time.

- Assume the government runs a "too large" welfare state.
- Given large insurance, people hold few assets.
- Reducing the welfare state in t_0 leads to too large risk for households on the transition path.
- These transition dynamics may be more important than (discounted) stationary equilibrium comparison.

Solving Transition Path: Aiyagari

We know value and policy function in old and new stationary equilibrium: $v^O(a,\epsilon), \phi^O(a,\epsilon)$ and $v^N(a,\epsilon), \phi^N(a,\epsilon)$. Assume new equilibrium is reached in T.

What happens in between? Assume $K^N = \int a_i d\lambda^N(a,\epsilon) > K^O = \int a_i d\lambda^O(a,\epsilon)$.

Solution solves:

$$c_t^{-\gamma} = \beta r_{t+1} \mathbb{E} c_{t+1}^{-\gamma}$$
$$c_t + a_{t+1} = (1 + r_t) a_t + w_t \epsilon$$
$$a_{t+1} \geq \underline{a}$$

To solve this in t, I need: r_t, r_{t+1}, w_t.

$$\Longrightarrow$$

I need $K_{t=0:T}$ with $K_0 = K^O$ and $K_T = K^N$.

Algorithm Solving Transition Path: Aiyagari

- Solve stationary equilibrium under old and new policy.
- Assume new stationary equilibrium is reached after T periods.
- Guess transition path for $K_{t=1:T-1}$. $w_t = F_L(K_t)$, $r_t = F_K(K_t)$.
- Given prices, solve household problem along the transition path.
- Compute $\hat{K}_t = \int a_i d\lambda_t(a, \epsilon)$ along transition path.
- If $|K_{t=1:T-1} - \hat{K}_{t=1:T-1}| > crit$ update initial guess.
- Iterate to convergence.

Back to Welfare

- The procedure gives two value function:

  v^O when no reform is passed.

  v^1 when reform is passed (with transition path).

$$\omega^U = \left(\frac{\int v^1(a,\epsilon) d\lambda_0(a,\epsilon)}{\int v^0(a,\epsilon) d\lambda_0(a,\epsilon)} \right)^{\frac{1}{1-\gamma}} - 1$$

References

Flodén, M. (2001): "The Effectiveness of Government Debt and Transfers as Insurance," *Journal of Monetary Economics*, 48, 81–108.